



زتونة المجالات

Vector Calculus

Convert Between [POINTS]

- Cylindrical to Cartesian

$$x = \rho \cos \phi \quad ; \quad y = \rho \sin \phi \quad ; \quad z = z$$

- Cartesian to Cylindrical

$$\rho = \sqrt{x^2 + y^2} \quad ; \quad \phi = \tan^{-1} \left(\frac{y}{x} \right) \quad ; \quad z = z$$

- Spherical to Cartesian

$$x = r \sin \theta \cos \phi \quad ; \quad y = r \sin \theta \sin \phi \quad ; \quad z = r \cos \theta$$

- Cartesian to Spherical

$$r = \sqrt{x^2 + y^2 + z^2} \quad ; \quad \phi = \cos^{-1} \left(\frac{z}{r} \right) \quad ; \quad \theta = \tan^{-1} \left(\frac{y}{x} \right)$$

Convert Between [Vectors]

Cartesian-Cylindrical

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos \phi$	$-\sin \phi$	0
$\mathbf{a}_y \cdot$	$\sin \phi$	$\cos \phi$	0
$\mathbf{a}_z \cdot$	0	0	1

Cartesian - Spherical

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin \theta \cos \phi$	$\cos \theta \cos \phi$	$-\sin \phi$
$\mathbf{a}_y \cdot$	$\sin \theta \sin \phi$	$\cos \theta \sin \phi$	$\cos \phi$
$\mathbf{a}_z \cdot$	$\cos \theta$	$-\sin \theta$	0

Scalar triple product	Vector triple product
$\bar{\mathbf{A}} \cdot (\bar{\mathbf{B}} \times \bar{\mathbf{C}}) = \begin{vmatrix} x_A & y_A & z_A \\ x_B & y_B & z_B \\ x_C & y_C & z_C \end{vmatrix}$ تمثل مساحة متوازي المستطيلات لو بتساوي صفر انن هم على مستوى واحد $\bar{\mathbf{A}} \cdot (\bar{\mathbf{B}} \times \bar{\mathbf{C}}) = \bar{\mathbf{B}} \cdot (\bar{\mathbf{C}} \times \bar{\mathbf{A}}) = \bar{\mathbf{C}} \cdot (\bar{\mathbf{A}} \times \bar{\mathbf{B}})$	$\bar{\mathbf{A}} \times (\bar{\mathbf{B}} \times \bar{\mathbf{C}})$ Lagrange's formula $\bar{\mathbf{A}} \times (\bar{\mathbf{B}} \times \bar{\mathbf{C}}) = \bar{\mathbf{B}}(\bar{\mathbf{A}} \cdot \bar{\mathbf{C}}) - \bar{\mathbf{C}}(\bar{\mathbf{A}} \cdot \bar{\mathbf{B}})$ Jacobi identity $\bar{\mathbf{A}} \times (\bar{\mathbf{B}} \times \bar{\mathbf{C}}) + \bar{\mathbf{B}} \times (\bar{\mathbf{C}} \times \bar{\mathbf{A}}) + \bar{\mathbf{C}} \times (\bar{\mathbf{A}} \times \bar{\mathbf{B}}) = 0$ $(\bar{\mathbf{A}} \times \bar{\mathbf{B}}) \times \bar{\mathbf{C}} = \bar{\mathbf{A}} \times (\bar{\mathbf{B}} \times \bar{\mathbf{C}}) - \bar{\mathbf{B}} \times (\bar{\mathbf{A}} \times \bar{\mathbf{C}})$

Gradient theorem	$\int_a^b \nabla T \cdot d\vec{l} = T(b) - T(a)$
Divergence theorem (Green , Gauss)	$\int_{\text{volume}} (\nabla \cdot \vec{V}) dv = \oint_{\text{surface}} \vec{V} \cdot d\vec{s}$
Curl Theorem (Stoke)	$\int_{\text{surface}} (\nabla \times \vec{V}) \cdot d\vec{s} = \oint_{\text{line}} \vec{V} \cdot d\vec{l}$

Differential Element		
Cartesian	Cylindrical	Spherical
$dl = dx\vec{a}_x + dy\vec{a}_y + dz\vec{a}_z$ $d\vec{A}_x = dydz\vec{a}_x$ $d\vec{A}_y = dzdx\vec{a}_y$ $d\vec{A}_z = dxdy\vec{a}_z$ $dV = dxdydz$	$dl = d\rho\vec{a}_\rho + \rho d\phi\vec{a}_\phi + dz\vec{a}_z$ $d\vec{S}_x = \rho d\phi dz\vec{a}_\rho$ $d\vec{S}_y = dzd\rho\vec{a}_\phi$ $d\vec{S}_z = \rho d\rho d\phi\vec{a}_z$ $dV = \rho d\rho d\phi dz$	$dl = dr\vec{a}_r + r d\theta\vec{a}_\theta + r \sin\theta d\phi\vec{a}_\phi$ $d\vec{S}_r = r^2 \sin\theta \cdot d\theta d\phi\vec{a}_r$ $d\vec{S}_\theta = r \sin\theta \cdot dr d\theta\vec{a}_\theta$ $d\vec{S}_\phi = r dr d\theta\vec{a}_\phi$ $dV = r^2 \sin\theta dr d\theta d\phi$

Gradience (scalar) :

$$\nabla f = \left(\frac{\partial f}{\partial x} \vec{a}_x + \frac{\partial f}{\partial y} \vec{a}_y + \frac{\partial f}{\partial z} \vec{a}_z \right)$$

Div Laws (Vector)

$$\text{div } \mathbf{D} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \right) \quad (\text{rectangular})$$

Curl

$$\nabla \times \mathbf{A} = \begin{bmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x_A & y_A & z_A \end{bmatrix}$$

GRADIENT

RECTANGULAR $\nabla V = \frac{\partial V}{\partial x} \mathbf{a}_x + \frac{\partial V}{\partial y} \mathbf{a}_y + \frac{\partial V}{\partial z} \mathbf{a}_z$

CYLINDRICAL $\nabla V = \frac{\partial V}{\partial \rho} \mathbf{a}_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \mathbf{a}_\phi + \frac{\partial V}{\partial z} \mathbf{a}_z$

SPHERICAL $\nabla V = \frac{\partial V}{\partial r} \mathbf{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \mathbf{a}_\theta + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \mathbf{a}_\phi$

DIVERGENCE

RECTANGULAR $\nabla \cdot \mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$

CYLINDRICAL $\nabla \cdot \mathbf{D} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho D_\rho) + \frac{1}{\rho} \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$

SPHERICAL $\nabla \cdot \mathbf{D} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (D_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial D_\phi}{\partial \phi}$

CURL

RECTANGULAR $\nabla \times \mathbf{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \mathbf{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) \mathbf{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \mathbf{a}_z$

CYLINDRICAL $\nabla \times \mathbf{H} = \left(\frac{1}{\rho} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} \right) \mathbf{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} \right) \mathbf{a}_\phi$
 $+ \frac{1}{\rho} \left[\frac{\partial (\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi} \right] \mathbf{a}_z$

SPHERICAL $\nabla \times \mathbf{H} = \frac{1}{r \sin \theta} \left[\frac{\partial (H_\phi \sin \theta)}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi} \right] \mathbf{a}_r + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial H_r}{\partial \phi} - \frac{\partial (r H_\phi)}{\partial r} \right] \mathbf{a}_\theta$
 $+ \frac{1}{r} \left[\frac{\partial (r H_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta} \right] \mathbf{a}_\phi$

Symbol	Name	Unit	Abbreviation
v	Velocity	meter/second	m/s
F	Force	newton	N
Q	Charge	coulomb	C
r, R	Distance	meter	m
ϵ_0, ϵ	Permittivity	farad/meter	F/m
E	Electric field intensity	volt/meter	V/m
ρ_v	Volume charge density	coulomb/meter ³	C/m ³
v	Volume	meter ³	m ³
ρ_L	Linear charge density	coulomb/meter	C/m
ρ_S	Surface charge density	coulomb/meter ²	C/m ²
Ψ	Electric flux	coulomb	C
D	Electric flux density	coulomb/meter ²	C/m ²
S	Area	meter ²	m ²
W	Work, energy	joule	J
L	Length	meter	m
V	Potential	volt	V
p	Dipole moment	coulomb-meter	C·m
I	Current	ampere	A
J	Current density	ampere/meter ²	A/m ²
μ_e, μ_h	Mobility	meter ² /volt-second	m ² /V·s
e	Electronic charge	coulomb	C
σ	Conductivity	siemens/meter	S/m
R	Resistance	ohm	Ω
P	Polarization	coulomb/meter ²	C/m ²
$\chi_{e,m}$	Susceptibility		
C	Capacitance	farad	F
R_s	Sheet resistance	ohm per square	Ω
H	Magnetic field intensity	ampere/meter	A/m
K	Surface current density	ampere/meter	A/m
B	Magnetic flux density	tesla (or weber/meter ²)	T (or Wb/m ²)
μ_0, μ	Permeability	henry/meter	H/m
Φ	Magnetic flux	weber	Wb
V_m	Magnetic scalar potential	ampere	A
A	Vector magnetic potential	weber/meter	Wb/m
T	Torque	newton-meter	N·m
m	Magnetic moment	ampere-meter ²	A·m ²
M	Magnetization	ampere/meter	A/m
$\mathcal{R}$	Reluctance	ampere-turn/weber	A·t/Wb
L	Inductance	henry	H
M	Mutual inductance	henry	H

Symbol	Name	Unit	Abbreviation
ω	Radian frequency	radian/second	rad/s
c	Velocity of light	meter/second	m/s
λ	Wavelength	meter	m
η	Intrinsic impedance	ohm	Ω
k	Wave number	meter ⁻¹	m ⁻¹
α	Attenuation constant	neper/meter	Np/m
β	Phase constant	radian/meter	rad/m
f	Frequency	hertz	Hz
S	Poynting vector	watt/meter ²	W/m ²
P	Power	watt	W
δ	Skin depth	meter	m
Γ	Reflection coefficient		
s	Standing wave ratio		
γ	Propagation constant	meter ⁻¹	m ⁻¹
G	Conductance	siemen	S
Z	Impedance	ohm	Ω
Y	Admittance	siemen	S
Q	Quality factor		

رايى الخاص (ممكن بطلع غلط) اللي فاهم بيه طريقه ال Div وال Curl تخيل الأسهم هي حركة موجات ميه والنقطه اللي بتدرس عندها (او المركز لو محدش) فيها كورة بينج بول ، لو الكورة هتتحرك أي حته فوق تحت يمين شمال يبقى في Div موجود ، لو هتلف حوالين نفسها يبقى في curl لو هتتحرك وتلف حوالين نفسها الاتنين يبقى في الاتنين لو الموجات بتلاشي بعض وهتفضل ثابتة ومش هتتحرك يبقى مفيش حاجه

ELECTRIC FIELDS

Columbs Law :

$$\vec{F} = \frac{(Q_1 Q_2)}{4\pi\epsilon_0 R^2} \vec{a}_R = \frac{(Q_1 Q_2)}{4\pi\epsilon_0 R^3} \vec{R}$$

Electric Field Laws

Point Charge	Line charge	Infinite line	
$E = \frac{Q}{4\pi\epsilon_0 R^3} \vec{R}$	$E = \int \frac{\rho_L dL}{4\pi\epsilon_0 R^2} \vec{a}_R$	$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 R} \vec{a}_R$	
Ring	Disk	Surface	Infinite surface
$\frac{1}{4\pi\epsilon_0} \frac{\rho_L L x}{(x^2 + a^2)^{\frac{3}{2}}}$ $= \frac{\rho_0 a h}{2\epsilon_0 (a^2 + h^2)^{\frac{3}{2}}} \hat{a}_z$	$E = \frac{\rho_s}{2\epsilon_0} \left[1 - \frac{x}{\sqrt{x^2 + R^2}} \right]$	$E = \int \frac{\rho_s ds}{4\pi\epsilon_0 R^2} \vec{a}_R$	$E = \frac{\rho_s}{2\epsilon_0}$

Electric Flux – Flux density – gauss law

$$\psi = Q_{enc} = \oiint \vec{D} \cdot d\vec{s}$$

علاقه D بال E

$$\frac{D}{E} = \epsilon \quad \therefore \quad D = \frac{E}{\epsilon}$$

Electric Density (D) relations

Point Charge	Infinite line	Infinite surface	Cylindrical volume
$D = \frac{Q}{4\pi R^3} \vec{R}$	$\vec{D} = \frac{\rho_L}{4\pi R} \vec{a}_R$	$D = \frac{\rho_s}{2}$	$\rho < a$ $D = \frac{\rho_v}{2} \rho$ $\rho > a$ $\frac{\rho_v \left(\frac{a^2}{2}\right)}{\rho}$
Finite cylindrical surface	Finite spherical shell surface	Surface	
$D = \frac{\rho_s}{\rho} a$	$D = \rho_s \frac{a^2}{r^2}$	$E = \int \frac{\rho_s ds}{4\pi\epsilon_0 R^2} \vec{a}_R$	

Dot form of charge density

From green's or gauss theorem

$$\int_{\text{volume}} (\vec{\nabla} \cdot \vec{V}) dv = \oint_{\text{surface}} \vec{V} \cdot d\vec{s}$$

$$\therefore \oiint \vec{D} \cdot d\vec{s} = \int \vec{\nabla} \cdot \vec{D} dv = \int \rho_v dv$$

$$\therefore \vec{\nabla} \cdot \vec{D} = \rho_v \quad \text{Maxwell's 1st equation}$$

Energy and potential

$$W = - \int_{initial}^{final} q \bar{E} \cdot d\bar{L}$$

$$Note : W_{AB} = -W_{BA} \quad ; \quad W_{ABCD A} = 0$$

$$V_{AB} = - \int_B^A \bar{E} \cdot d\bar{L}$$

$$\oint \bar{E} \cdot d\bar{L} = 0$$

$$\bar{\nabla} \times \bar{E} = \mathbf{0}$$

Maxwell's 2nd equation (assuming conservative system)

$$\oint \bar{E} \cdot d\bar{L} = \bar{\nabla} \times \bar{E} \neq \mathbf{0} \text{ (non - conservative system)}$$

Potential difference Equations :

Point Charge	Absolute potential	Line charge	Infinite line
$V_{AB} = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_A} - \frac{1}{r_B} \right]$	$V_A = V_{A\infty} = \frac{Q}{4\pi\epsilon_0 r_A}$	$V = \int \frac{\rho_L dL}{4\pi\epsilon_0 r}$	$V_{AB} = \frac{\rho_L}{2\pi\epsilon_0} \ln \frac{\rho_B}{\rho_A}$

Equipotential surfaces : surfaces that has the same potential

$$\frac{\Delta V}{\Delta L} = \text{Potential gradient}$$

$$V = - \int \bar{E} \cdot d\bar{L}$$

$$\Delta V = -E \Delta L$$

كمعيار

$$\therefore E = - \frac{\Delta V}{\Delta L}$$

كمنتجه

$$\bar{E} = -\nabla V$$

To move a charge from infinity to a point in the free space with other charges affect that point with a field

$$W_E = \frac{1}{2} \sum_{m=1}^n Q_m V_m$$

$$W_E = \frac{1}{2} \int \bar{D} \cdot \bar{E} \, dV \quad (J)$$

$$W_E = \frac{1}{2} \bar{D} \cdot \bar{E} \quad \left(\frac{\rho}{m^3} \right)$$

Electrostatic Fields in materials

يمكننا تقسيم الشحنات لشحنه سطحيه و حجميه ، في الموصل نتعامل على ان السلك اسطوانه يمر بها شحنه حجميه
العوامل اللي بتكون التيار (كثافه الشحنات ، مساحه المقطع ، طول ، الزمن (سرعه الموصلات))

$$i = \frac{\rho_v \Delta S \Delta L}{\Delta t} \quad (\text{divide by area})$$

$$J = \rho_v v_d$$

لا تعتمد على نوع توصيليه ماده ، تعتمد على نوع التوصيل conduction , convention

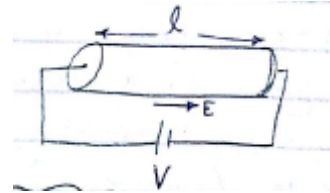
Ohm's Law

$$J = \sigma \bar{E} \quad (A.m)$$

$$V = -\int E \cdot dL = E \cdot L$$

$$\text{ohm's law } J = \sigma E \Rightarrow \frac{I}{S} = \sigma \frac{V}{L} \Rightarrow \frac{V}{I} = \frac{L}{\sigma A} = R$$

$$\text{for non uniform } V : R = \frac{V}{I} = \frac{-\int E \cdot dL}{\int_s E \cdot dS}$$



Field Effect on Dielectric Materials

يتم استقطاب ماده عند وجود مجال كهربي عليها

$$P = \chi_e \epsilon_0 \bar{E}$$

$$\chi_e = \epsilon_r - 1$$

اقل مجال كهربي يكسر العزم : Dielectric Strenght

$$D = \epsilon E = \epsilon_0 \epsilon_r E = \epsilon_0 (1 + \chi_e) E = P + \epsilon_0 E$$

خواص المواد العازلة : (لا يوجد عزل مثالي

الماده العازلة لا تحتوي الكترولونات حرة

عند التاثير على المواد العازلة يتم استقطابها

Boundary Conditions

Between 1- Dielectric – Dielectric 2- Dielectric –Conductor

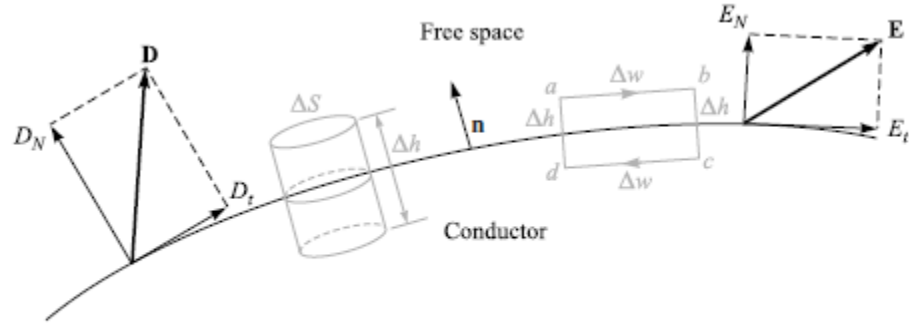


Figure 5.4 An appropriate closed path and gaussian surface are used to determine boundary conditions at a boundary between a conductor and free space; $E_t = 0$ and $D_N = \rho_s$.

1- Dielectric – Dielectric

Normal	Tangent
Gauss (الشحنه على السطح الفاصل بين المادتين ρ_s) $\oiint D \cdot dS = Q_{enc}$ $D_{1n} \Delta S - D_{2n} \Delta S = \rho_s \Delta S$ $D_{1n} - D_{2n} = \rho_s$ if $\rho_s = 0$ $\therefore D_{1n} = D_{2n}$	Ampere's law $\oint E \cdot dL = 0$ $\therefore E_{1t} \Delta w - E_{1n} \frac{\Delta h}{2} - E_{2n} \frac{\Delta h}{2} - E_{2t} \Delta w$ $+ E_{2n} \frac{\Delta h}{2} + E_{2n} \frac{\Delta h}{2} = 0$ $\therefore E_{1t} = E_{2t}$

2- Conductor – Dielectric

Normal	Tangent
Gauss $\oiint D \cdot dS = Q_{enc}$ $\therefore D_n = \rho_s$	$E_t = 0$

Capacitance

$$C = \frac{Q}{V} \left((if E \text{ (electric field is uniform)}) \right)$$

$$W_E = \frac{1}{2} C V^2$$

ملحوظات للمكثفات

1- هيحي مساله مكثفات وش

2- المكثفات على التوالي

$$C_t = \frac{C_1 C_2}{C_1 + C_2}$$

المكثفات على التوازي

$$C_t = C_1 + C_2$$

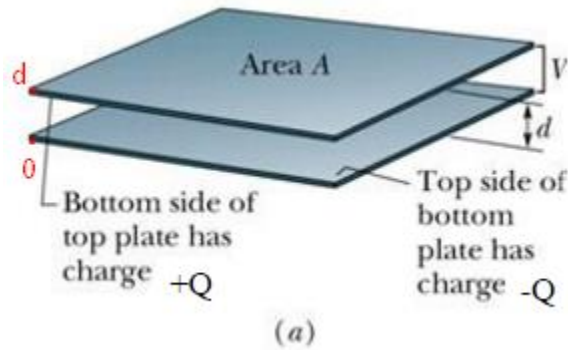
3- أنواع المكثفات ممكن يكون

Parallel plate- Coaxial (cylindrical) – Spherical

وركز في شكل المكثف والعدد في حاله اسطواني او كروي مش مستطيل

استنتاج قانون المكثف :

Parallel Plate :



$$C = \frac{Q}{V}$$

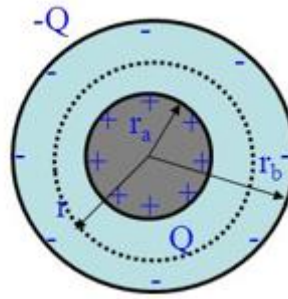
$$V = - \int_d^0 E \cdot dL$$

$$V = - \int_0^d \frac{\rho_s}{\epsilon} \cdot dL$$

$$\therefore V = \frac{\rho_s}{\epsilon} d$$

$$\therefore C = \frac{Q}{V} = \frac{\rho_s A}{\frac{\rho_s}{\epsilon} d} = \frac{\epsilon A}{d}$$

Spherical



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0} \rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

$$V_{ab} = V_a - V_b = \frac{Q}{4\pi\epsilon_0 r_a} - \frac{Q}{4\pi\epsilon_0 r_b} = \frac{Q}{4\pi\epsilon_0} \frac{r_b - r_a}{r_a r_b}$$

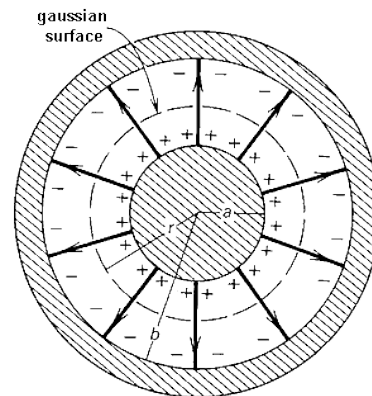
$$C = \frac{Q}{V_{ab}} = 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$$

Cylindrical (Co-axial)

$$C = \frac{Q}{V}$$

$$C = \frac{\lambda L}{\frac{\lambda}{2\pi\epsilon_0} \left(\ln \frac{b}{a} \right)}$$

$$\frac{C}{L} = \frac{2\pi\epsilon_0}{\left(\ln \frac{b}{a} \right)}$$



MAGNETIC FIELDS

The Sources of Magnetic Field :

1-Permanent magnet 2-Linearly changing electric field w/ time 3- a direct current

Biot-Savart

$$H \propto \frac{m_1 m_2}{R^2} \quad \text{القوة المؤثرة على وحده الاقطاب المغناطيسية}$$

$$d\vec{H} = \frac{I d\vec{L} \times \vec{a}_R}{4\pi R^2} = \frac{I d\vec{L} \times \vec{R}}{4\pi R^3}$$

The total current crossing any closed surface is zero. It is this current flowing in a closed circuit that must be our experimental source (In a closed path), not the differential element. It follows that only the integral form of the Biot-Savart law can be verified experimentally, $\vec{H} = \oint \left(\frac{I d\vec{L} \times \vec{a}_R}{4\pi R^2} \right)$

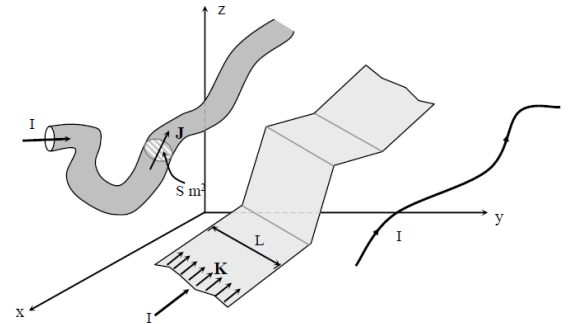
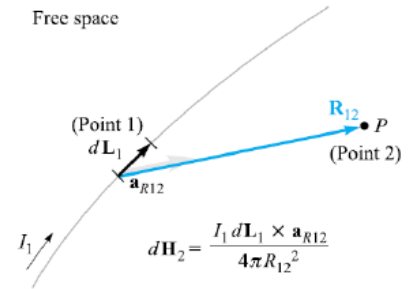
التيار الكهربائي هو اهم مصادر المجال المغناطيسي عشان كده لازم ندرس اشكاله المختلفه

Surface current density : $I = K * L$ (uniform) or $I = \int K dl$

the same with

$$I d\vec{L} = K d\vec{s} = J d\vec{V}$$

$$H = \int_S \frac{K d\vec{s} \times \vec{a}_R}{4\pi R^2} = \int_V \frac{J d\vec{v} \times \vec{a}_R}{4\pi R^2}$$



H from :

Infinite Line	Line
$\vec{H} = \frac{I}{2\pi\rho} \vec{a}_\phi$	$\vec{H} = \frac{I}{4\pi\rho} [\sin(\alpha_2) - \sin(\alpha_1)] \vec{a}_\phi$

H from Coaxial cable

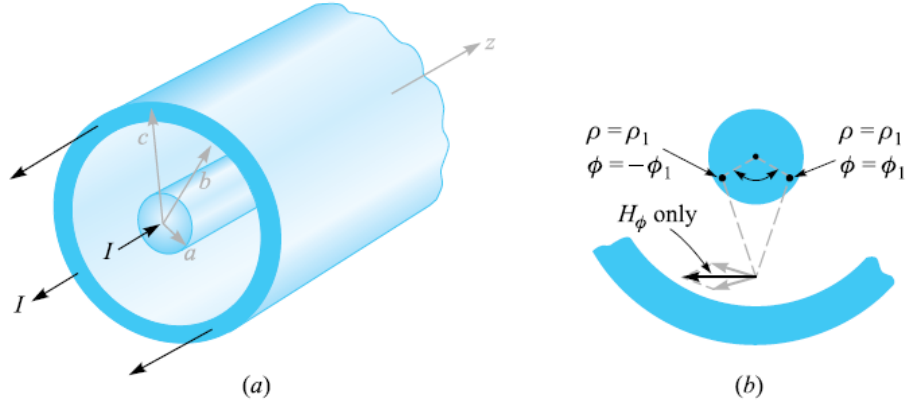


Figure 7.8 (a) Cross section of a coaxial cable carrying a uniformly distributed current I in the inner conductor and $-I$ in the outer conductor. The magnetic field at any point is most easily determined by applying Ampère's circuital law about a circular path. (b) Current filaments at $\rho = \rho_1$, $\phi = \pm\phi_1$, produces H_ϕ components which cancel. For the total field, $H = H_\phi \mathbf{a}_\phi$.

بتطبيق قانون امبير (شبه قانون جاوس في الكهربائية)

$\rho < a$	$a < \rho < b$	$b < \rho < c$	$\rho > c$
$I_{\text{encl}} = I \frac{\rho^2}{a^2}$ $2\pi\rho H_\phi = I \frac{\rho^2}{a^2}$ $H_\phi = \frac{I\rho}{2\pi a^2} \quad (\rho < a)$	$H_\phi = I$	$2\pi\rho H_\phi = I - I \left(\frac{\rho^2 - b^2}{c^2 - b^2} \right)$ $H_\phi = \frac{I}{2\pi\rho} \frac{c^2 - \rho^2}{c^2 - b^2} \quad (b < \rho < c)$	$H_\phi = 0 \quad (\rho > c)$

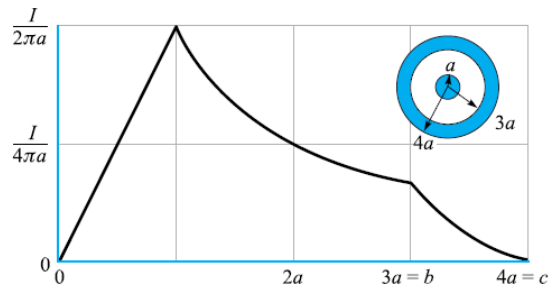
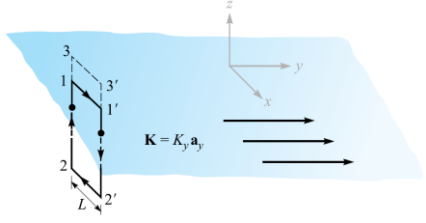
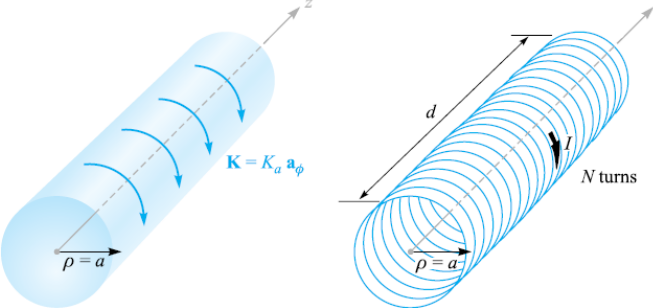
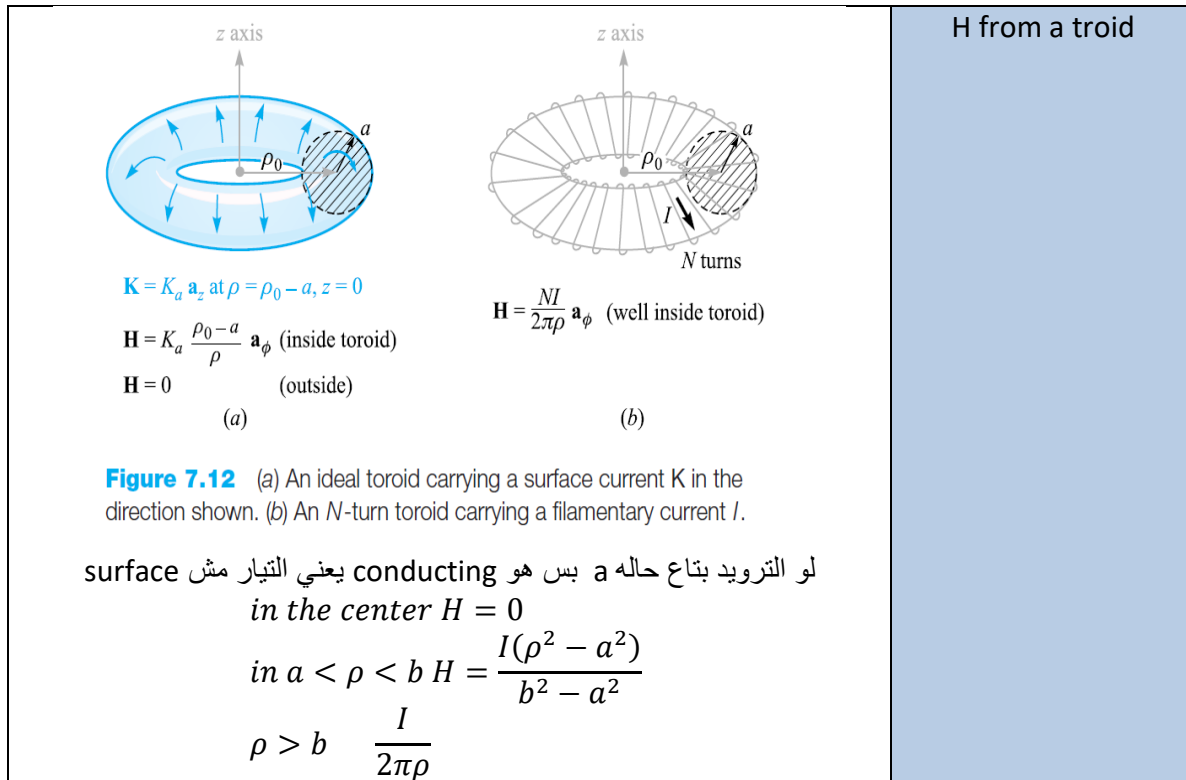


Figure 7.9 The magnetic field intensity as a function of radius in an infinitely long coaxial transmission line with the dimensions shown.

لاي نقطه على احد الجانبين $\mathbf{H} = \frac{1}{2} \mathbf{K} \times \mathbf{a}_N$ 	H from a surface
 $\mathbf{H} = K_a \mathbf{a}_z, \rho < a$ $\mathbf{H} = 0, \rho > a$ (a) $\mathbf{H} = \frac{NI}{d} \mathbf{a}_z$ (well inside coil) (b) Figure 7.11 (a) An ideal solenoid of infinite length with a circular current sheet $\mathbf{K} = K_a \mathbf{a}_\phi$. (b) An N-turn solenoid of finite length d. لو محدود $H = \frac{nI}{2} [\cos \theta_2 - \cos \theta_1]$	H from a solenoid



Force from a Magnetic Field and Electric Field

As shown from the direction of $\mathbf{H}$ the Force from a magnetic field is in Right عموديه angle with the velocity of the particle

Which means : Magnetic Force can never change the velocity of a moving particle ; this means that magnetic field is incapable of transferring Energy to the moving charge

$$F_e(\text{Electric Force}) = Q\bar{E}$$

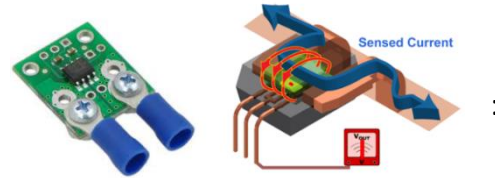
$$F_m(\text{Magnetic Force}) = Q \bar{V} \times \bar{B}$$

$$F_{total}(\text{Superposition force}) = Q(\bar{E} + \bar{V} \times \bar{B})$$

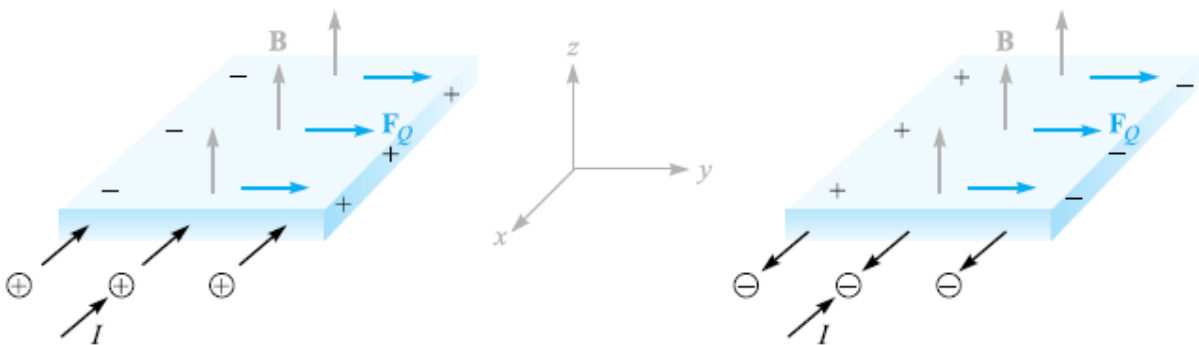
Notes : a large number of very small forces $dF = dQ V \times B$ can cause electrons to shift slightly and then the conductor as a whole , it causes displacement in center of gravity between +ve and -ve charges so the columb (electric) force tend to resist this displacement>

What's Current sensor (Hall Voltage)

It's a method to differentiate between the electrons and holes current by comparing their hall voltage in what's called current sensor this helps to determine if the semiconductor is n-type or p-type



The charge separation that does result, however, is disclosed by the presence of a slight potential difference across the conductor sample in a direction perpendicular to both the magnetic field and the velocity of the charges. The voltage is known as the Hall voltage, and the effect itself is called the Hall effect.



Force on a current-carrying conductor

$$J = \rho_v \bar{v} \text{ and } Q = \rho_v dV$$

$$dF = dQ \bar{v} \times \bar{B} = \rho_v dV \frac{J}{\rho_v} \times \bar{B} = J d\bar{V} \times \bar{B} = (\bar{J} \times \bar{B}) dV$$

$$\text{the same } dF = (\bar{J} \times \bar{B}) dV = (\bar{K} \times \bar{B}) dS = (\bar{I} \times \bar{B}) dL = I d\bar{L} \times \bar{B}$$

$$\bar{F} = \int_v (\bar{J} \times \bar{B}) dV = \int_s (\bar{K} \times \bar{B}) dS$$

$$\bar{F} = \oint I d\bar{L} \times \bar{B} = -I \oint \bar{B} \times d\bar{L} = I \bar{L} \times \bar{B}$$

$$|F| = BIL \sin(\theta)$$

We can express Force between 2 current elements without the magnetic field but it will be complex as follows:

$$\text{from : } dF = I_2 d\vec{L}_2 \times \vec{B} \text{ \& } d\vec{H} = \frac{I_1 d\vec{L}_1 \times \vec{a}_R}{4\pi R^2} \text{ \& } B = \mu_0 H$$

$$\therefore F_2 = \mu_0 \frac{I_1 I_2}{4\pi} \oint [d\vec{L}_2 \times \oint \frac{d\vec{L}_1 \times \vec{a}_R}{R^2}] \rightarrow (\text{very very complex})$$

NOTE : the total force on a closed current loop in a uniform = 0

TORQUE

$$\vec{T} = \vec{R} \times \vec{B}$$

For a closed loop

$$d\vec{T} = I d\vec{S} \times \vec{B}$$

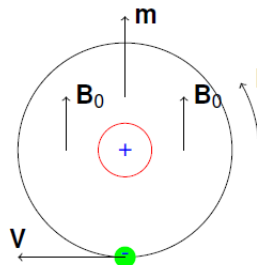
$$\text{define } d\vec{m} = I d\vec{S}$$

$$\therefore d\vec{T} = d\vec{m} \times \vec{B} \quad (\text{dipole moment } A.m^2)$$

$$\vec{T} = \vec{m} \times \vec{B} = I \vec{S} \times \vec{B}$$

The torque direction is that to align the magnetic field produced by the loop with the applied magnetic field causing the torque

في محاوله هنعتبر الالكترن حوالين النواه يمثل لوب تياره عكس اتجاه حركه الالكترن وتم تسليط فيض مغناطيسي خارجي ده هيسبب ان التيار بتاع حركه الالكترونات الصغير ده هيتولد مجال مغناطيسي يحاول يحرك النواه في اتجاه المجال الخارجي لحد ما يبقوا aligned ويساعدو بعض - او يضادوا ! - على حسب



لهذا السبب

1- المجال المغناطيسي داخل المادة هيكون اكبر منه خارج المادة بسبب ان الالكترونات هتولد مجال إضافي داخلها

2- ال spin magnetic moment يتناح الإلكترون بتساوي $9 \times 10^{-24} Am^2 \pm$ موجب لو بتساعد سالب لو بتضاد المجال الخارجي

3- الإلكترونات اللي بتساهم بمجال مغناطيسي فقط الإلكترونات اللي في المدارات الخارجية غير المكتملة
4- طبعا كل الكلام اللي فوق ده محتاج معادلات من ميكانيكا الكم و حورات المهم اللي نستنتج ان نوع المادة وعدد الإلكترونات و اللي بنكلم فيه ده جزء صغير جدا لان المادة مليانه moments لاسباب مختلفه وكلها بتاثر على تصنيف المادة المغناطيسيه الا ان التبسيط اللي فوق ده هو الأساس العلمي لاجهزه ال MRI ربنا لا يدخلكم في واحده

Types Of Material

يمكن تصنيف المواد على حسب اذا كانت المومنت المحصله للإلكترونات بتزود الفيض داخلها ام تقلله ام ماذا

Table 8.1 Characteristics of magnetic materials

Classification	Magnetic Moments	B Values	Comments
Diamagnetic	$\mathbf{m}_{orb} + \mathbf{m}_{spin} = 0$	$B_{int} < B_{appl}$	$B_{int} \doteq B_{appl}$
Paramagnetic	$\mathbf{m}_{orb} + \mathbf{m}_{spin} = \text{small}$	$B_{int} > B_{appl}$	$B_{int} \doteq B_{appl}$
Ferromagnetic	$ \mathbf{m}_{spin} \gg \mathbf{m}_{orb} $	$B_{int} \gg B_{appl}$	Domains
Antiferromagnetic	$ \mathbf{m}_{spin} \gg \mathbf{m}_{orb} $	$B_{int} \doteq B_{appl}$	Adjacent moments oppose
Ferrimagnetic	$ \mathbf{m}_{spin} \gg \mathbf{m}_{orb} $	$B_{int} > B_{appl}$	Unequal adjacent moments oppose; low σ
Superparamagnetic	$ \mathbf{m}_{spin} \gg \mathbf{m}_{orb} $	$B_{int} > B_{appl}$	Nonmagnetic matrix; recording tapes

اذن يمكننا تعريف 3 تيارات

1- I وهو التيار الذي نعرفه ناتج عن حركه الإلكترونات حره في موصل وفي قانون امبير $\oint H \cdot dL = I$

2- التيار الناتج عن ال Bound charge والذي يعرف I_B وناتج عن المغناطيسية $\oint M \cdot dL = I_B$

3- التيار الكلي وهو مجموع $I_T = I_B + I$ ومن قانون امبير $\oint \left(\frac{B}{\mu_0}\right) \cdot dL = I_T$

منها نستنتج عده قوانين هامه

$$\frac{B}{\mu_0} = M + H \quad \therefore B = \mu_0 (H + M)$$

$$\oint M \cdot dL = I_B = \int_S J_B ds \quad \therefore \bar{\nabla} \times \bar{M} = \bar{J}_B$$

$$\oint \mathbf{H} \cdot d\mathbf{L} = I = \int_S \mathbf{J} \cdot d\mathbf{s} \quad \therefore \quad \nabla \times \mathbf{H} = \mathbf{J}$$

$$\oint \left(\frac{\mathbf{B}}{\mu_0} \right) \cdot d\mathbf{L} = I_T = \int_S \mathbf{J}_T \cdot d\mathbf{s} \quad \therefore \quad \nabla \times \frac{\mathbf{B}}{\mu_0} = \mathbf{J}_T$$

$$\text{where } \bar{M} = \text{atoms}/m^3 * (\text{dipole moment /atom}) = \frac{B}{\mu_0} \left(\frac{\chi_m}{1 + \chi_m} \right)$$

For Linear isotropic material (if χ_m magnetic susceptibility) $M = \chi_m H$
 $\therefore B = \mu_0 (H + \chi_m H) = \mu_0 (1 + \chi_m) H = \mu_0 \mu_r H \quad \text{where } \mu_r = (1 + \chi_m)$
define $\mu = \mu_r \mu_0$
then **$\mathbf{B} = \mu \mathbf{H}$**

Generally :

As,

$$\mathbf{B} = \mu \mathbf{H}$$

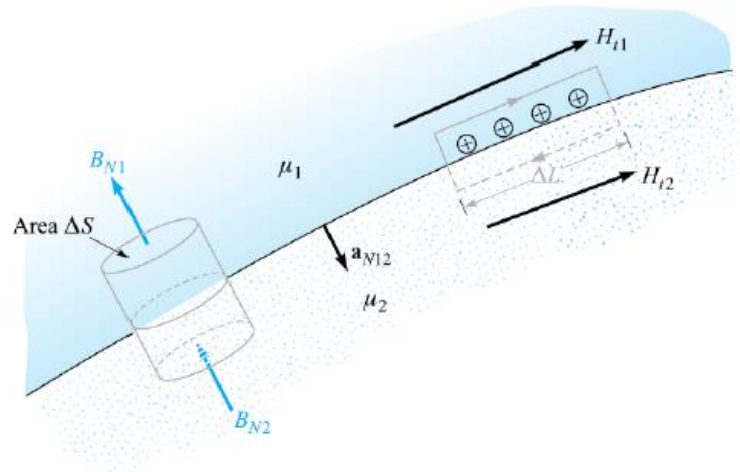
for homogeneous, linear, isotropic magnetic material that may be described in terms of a relative permeability μ_r

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \mu \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}$$

For anisotropic materials

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}$$

Boundary Conditions

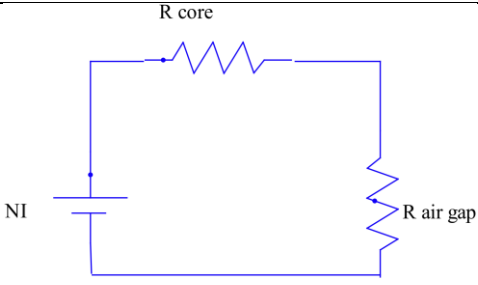
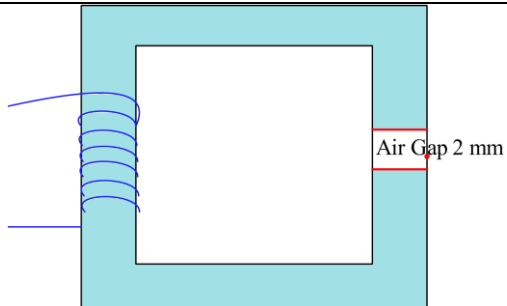


Normal	Tangent
$B_{N1} = B_{N2}$ $\mu_1 H_{N1} = \mu_2 H_{N2}$ $\frac{\mu_1 M_{N1}}{\chi_{m1}} = \frac{\mu_2 M_{N2}}{\chi_{m2}}$	$H_{t1} - H_{t2} = K$ $(H_{t1} - H_{t2}) \times a_{N12} = K$ $(H_{t1} - H_{t2}) = K \times a_{N12}$ $\frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = K$ $\frac{M_{t1}}{\chi_{m1}} - \frac{M_{t2}}{\chi_{m2}} = K$ Where K (surface current) K = 0 if either materials is conductor

MAGNETIC CIRCUIT

طبعاً الجزئية دي احنا فاهمين تطبيقها من التحويل لكن الأساس الفيزيائي العلمي واضح من مقارنه المعادلات بين الكهربية والمغناطيسية واللي ببساعدنا في تقريب الدائرة المغناطيسية لدائرة كهربية ذات تيار مستمر ومقاومات فقط

MAGNETIC	ELECTRIC
$1- H = -\nabla V_m$ $2- V_{mAB} = \int_A^B H \cdot dL$ $3- B = \mu H$ $4- \phi = \int B \cdot ds$ $5- V_m = \phi \mathfrak{R}$ $6- \mathfrak{R} = \frac{d}{\mu A}$ $7- \oint E \cdot dL = 0$ $8- W = \frac{1}{2} \int_v \bar{D} \cdot \bar{E} \, dv$	$1- E = -\nabla V$ $2- V_{AB} = \int_A^B E \cdot dL$ $3- J = \sigma \bar{E}$ $4- I = \int J \cdot dS$ $5- V = IR$ $6- R = \frac{d}{\sigma A}$ $7- \oint H \cdot dL = NI$ $8- W = \frac{1}{2} \int_c \bar{B} \cdot \bar{H} \, dv$ $W = \frac{1}{2} \int_v \mu H^2 \, dv = \frac{1}{2} \int_v \frac{B^2}{\mu} \, dv$

Electrical	Magnetic
$Emf = V = IR$	$Mmf = NI = \phi R = Hl$
$R = \frac{l}{\sigma A}$	$R = \frac{l}{\mu A}$ $\mu = \mu_0 \mu_r$ $\mu_0 = 4\pi \times 10^{-7}$
	

Inductance

Flux Linkage (λ) : the total flux linking the turns of a coil $\lambda = N\phi$

المفروض عند حساب ال flux linkage نحسب فيض كل لفه الأول بس بدل ما نعمل كده بنستخدم ال winding factor and pitch factor لتصحيح الخطأ في استخدام المعادله البسيطة

Inductance (L): the ration between the flux linkage and the current it's linking

ويزيد الحث الذاتي كلما زاد عدد اللفات او الفيض وحدتها ال H .

Inductance For a coaxial cable

$$a < \rho < b$$

$$H = \frac{I}{2\pi\rho} \Rightarrow B = \mu_0 H \Rightarrow \phi = \int_s B \cdot ds$$

$$\phi = \int_0^d \int_a^b \frac{\mu_0 I}{2\pi\rho} d\rho dz \quad a_\phi = \frac{\mu_0 I d}{2\pi} \ln \frac{b}{a}$$

$$L = \frac{N\phi}{I} = (\text{per meter lenght}) = \frac{\mu_0}{2\pi} \ln \frac{b}{a}$$

Mutual Inductance

The mutual flux linking two coils

At 90° between coils it is $M = 0$

At 0° between coils it is $M_{\max}$

$$M_{12} = M_{21} = \frac{N_1 \phi_{12}}{I_2} = \frac{N_2 \phi_{21}}{I_1} \quad (H)$$

Time Varying fields

As we know magnetic field varying with time can produce current

$$emf = -N \frac{d\phi}{dt}$$

(the -ve means that the flux act to produce an opposing flux (lenz' s law))

We can have $\frac{d\phi}{dt}$ by

- 1- A time-varying flux with a stationary current path
- 2- A relative motion between the a steady flux and closed path
- 3- Combination of the two

Kirchoff's Law

زمان أيام السيركتس والبراءة كنا بنقول ان كيرشوف بيقول ان محصله الجهود على مسار مغلق بتساوي صفر

$$\oint E \cdot dL = 0$$

لكن في حالة تولد emf مستحثه معدش $= 0$ لان هيظهر عندي نوعين من الجهد

$$-1 \quad \text{ناتج عن تغير الفيض مع الزمن} \quad -\frac{d\phi}{dt} = -\frac{d}{dt} \int_s B \cdot ds$$

$$-2 \quad \text{ناتج عن الحركة} \quad \frac{F}{Q} = E_m = v \times B$$

لذا

$$emf = \oint E \cdot dL = \int_s -\frac{\partial B}{\partial t} ds = \oint (\bar{v} \times \bar{B}) d\bar{L}$$

$$\text{transform voltage only } emf = \oint E \cdot dL = \int_s (\bar{\nabla} \times \bar{E}) ds = \int_s -\frac{\partial B}{\partial t} ds$$

$$\therefore \bar{\nabla} \times \bar{E} = -\frac{\partial B}{\partial t}$$

$$\text{Motional voltage only } emf = \oint E \cdot dL = \oint (\bar{v} \times \bar{B}) d\bar{L}$$

Displacement Current

زمان أيام السيركتس والبراءة كنا بنقول ان المكثف مبيمرش فيه تيار وكنا بنقول ان قانون امبير بسيط وبيقولك

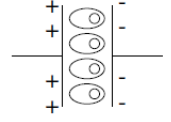
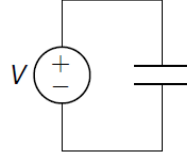
$$\oint H \cdot dl = I \quad \text{or} \quad \nabla \times H = J$$

لكن للتجارب العمليه في ال timed varying fields كان في جزء ناقص من المعادله دي وهو ال Displacement Current I_d

$$\oint H \cdot dl = I + I_d = I + \int_s \frac{dD}{dt} \cdot ds \quad \text{or} \quad \nabla \times H = J + J_d = J + \frac{dD}{dt}$$

وممكن نستنتج المقدار ده $\frac{dD}{dt}$ كالاتي

$$\begin{aligned} C &= \frac{\epsilon A}{d} ; V = \int E \cdot dL = Ed \\ \therefore I &= JA = C \frac{dv}{dt} = \frac{\epsilon A}{d} * d * \frac{dE}{dt} = \epsilon \frac{dE}{dt} * A = \frac{dD}{dt} * A \\ \therefore J_d &= \frac{dD}{dt} = \epsilon \frac{dE}{dt} \end{aligned}$$



MAXWELL EQUATIONS

Differential Form	Integral Form
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot d\vec{L} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\oint \vec{H} \cdot d\vec{L} = I + \int_s \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S}$
$\nabla \cdot \vec{D} = \rho$	$\oint_s \vec{D} \cdot d\vec{s} = \int_v \rho_v \cdot dv$
$\nabla \cdot \vec{B} = 0$	$\oint_s \vec{B} \cdot d\vec{s} = 0$

Auxiliary Equations :

$$\begin{array}{lll} \vec{D} = \epsilon \vec{E} & \vec{D} = \epsilon_0 \vec{E} + \vec{P} & \vec{P} = \chi_e \epsilon_0 \vec{E} \\ \vec{B} = \mu \vec{H} & \vec{B} = \mu_0 (\vec{H} + \vec{M}) & \vec{M} = \chi_m \vec{H} \\ \vec{J} = \sigma \vec{E} & \vec{J} = \rho_v \vec{V} & \end{array}$$

And God said,
" $\nabla \cdot \mathbf{D} = \rho$
 $\nabla \cdot \mathbf{B} = 0$
 $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
 $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$ "
and there was light.